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# Comparison of Extrusion Strains Produced by Cosine and Conical Dies

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**Visioplastic analysis is used to compare strain fields produced in cylindrical aluminum billets extruded at elevated temperature through axially symmetric dies that have either a cosine or a conical profile. The visioplastic method consists of computing strain rates from changes experienced in a flow function as a billet passes through the extrusion die. The flow function is constructed using a grid that is stamped on an axial plane of the billet before extrusion. After the partially extruded billet is removed from the die, numerical methods are used to determine the strain rates from the deformed grid. The state of strain over the extrusion region is computed using transformation and then integration of the strain rates. Results show that, unlike the conical die, the cosine die does not produce strains of reversed sign in the die entry region. The consequent benefits of using cosine dies for extrusion of powder metals are discussed.**

**Keywords**
extrusion, strain, powder metals

## 1. Introduction

DURING hot extrusion, material is heated and forced through a die. Conical dies are frequently used to reduce the cross section of extrusion billets to either rods or tubes. Billets are sometimes made from cast alloy powder or from powder metallurgical (P/M) blends. Alloyed powders are extruded to manufacture aluminum-lithium components for aircraft applications, and aluminum and uranium oxide powder blends are extruded to make tubes for nuclear reactor fuel elements.

Extruded aluminum P/M products tend to fail in the plastic region by separation of individual particles or ligaments (extruded particles). Mechanical testing of powder aluminum test specimens has revealed a behavior related to the particulate structure. Uniaxial tensile tests have shown that the plastic stress decreases with increasing plastic strain, producing a negative strain-hardening coefficient. This effect is reduced in biaxial loading, where a compressive transverse stress is placed on the specimen.

When material is extruded, a three-dimensional strain field is produced throughout the die region. The various strains may be tensile or compressive and are a function of the die shape. An investigation was performed using visioplastic strain analysis to examine strain distributions produced by a cosine-shaped axially symmetric die. These strains were compared with those produced by a conical die to examine differences that may result in improved product quality for cosine die extrusions.

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## 2. Cosine Die

A conical extrusion die reduces the diameter of a cylindrical billet by passing the heated material through a simple conical tapered region that serves as transition between the original entry diameter and the final exit diameter. The cosine die replaces the conical taper with a cosine profile that is revolved about the die axis. For the work performed in this paper, the radius, $R$ (Fig. 1), is given by:

$$R = 1.59 \cos(0.618x) + 2.86 \text{ cm}$$

It is revolved 360° about the $z$ (die) axis. The entry diameter is 8.9 cm, and the exit diameter is 2.5 cm. The extrusion ratio is 12 to 1. Profiles of conical and cosine dies are shown in Fig. 1.

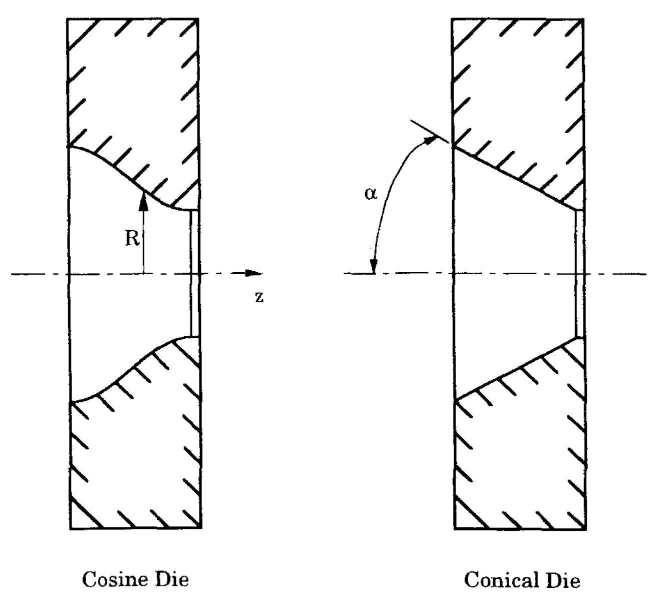


**Fig. 1** Diametral sections of cosine and conical dies

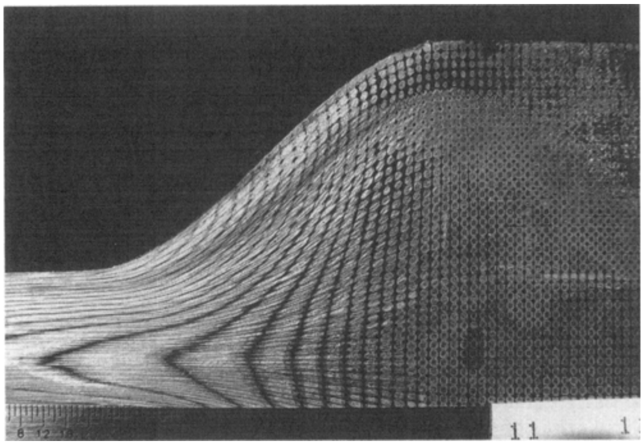

**Fig. 2** Partially extruded billet using the cosine die. Because of its greater clarity, the portion of the billet below the $z$ axis (Fig. 1) was used. It is shown inverted. The distance between "8" and "12" at the lower left is 3.2 mm (⅛ in.).

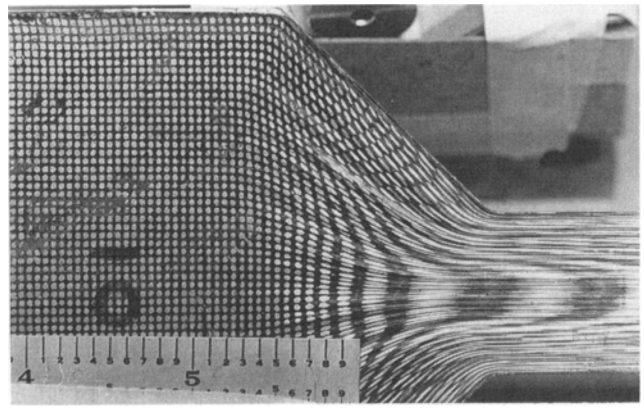

**Fig. 3** Partially extruded billet using the conical die. The distance between "4" and "5" is 25 mm (1 in.).

The cosine die is a contour die. For axisymmetric extrusion, contour die profiles have been examined experimentally and theoretically in attempts to reduce extrusion ram loading and understand surface effects and modify strain rates during extrusion (Ref 1, 2). Several studies have investigated contour dies applied to extrusion of nonsymmetric shapes to improve metal flow and product uniformity (Ref 3-7). In this paper, experimental visioplastic analysis is used to show how the cosine die improves distribution of the extrusion strain.

## 3. Visioplastic Strain Analysis

Visioplasticity is an experimental-numerical procedure used to obtain strain results. The method has been used for extrusion analysis with conical dies (Ref 8-10) and will be briefly reviewed here. Visioplasticity obtains strain values by analyzing the deformation that occurs in a rectangular grid stamped on the axial plane of a split, preextruded billet. The reassembled billet is pushed partway through the extrusion die and is then removed and opened to display the deformed grid. The grid lines that originally were parallel to the billet axis become laminar flowlines during extrusion.

For the axisymmetric flow, a flow function, $\phi$, is identified:

$$\phi(r, z) = 2\pi \int_0^r \rho v(\rho, z)\, d\rho \qquad \text{(Eq 1)}$$

where $r$ is the radius from the billet axis to a specific flowline, and $v$ is the velocity in the axial direction. Axial and radial velocity components can then be computed:

$$v(r, z) = \frac{1}{2\pi r}\frac{\partial \phi}{\partial r} \qquad u(r, z) = \frac{-1}{2\pi r}\frac{\partial \phi}{\partial z}$$

where $u$ is the radial velocity component.

Strain rates are computed:

$$\dot{\varepsilon}_r = \frac{\partial u}{\partial r} = \frac{1}{2\pi r}\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \frac{\partial^2 \phi}{\partial r \partial z}\right) \qquad \text{(Eq 2)}$$

$$\dot{\varepsilon}_z = \frac{\partial v}{\partial z} = \frac{1}{2\pi r}\frac{\partial^2 \phi}{\partial r \partial z} \qquad \text{(Eq 3)}$$

$$\dot{\varepsilon}_\theta = \frac{u}{r} = \frac{-1}{2\pi r^2}\frac{\partial \phi}{\partial z} \qquad \text{(Eq 4)}$$

$$\dot{\gamma}_{rz} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} = \frac{1}{2\pi}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial r}\right) - \frac{1}{r}\frac{\partial^2 \phi}{\partial z^2}\right] \qquad \text{(Eq 5)}$$

Because of axial symmetry, there is only one shear strain rate. To this point the method is similar to the velocity field method, which has been used to theoretically determine strain rates from an imposed velocity field for extrusion of a noncircular shape (Ref 5).

To obtain the strains explicitly, the strain rates must first be transformed to a set of curvilinear coordinates that are parallel and perpendicular to the flowlines. The only strain rate unaffected by the transformation is $\dot{\varepsilon}_\theta$. The transformation is necessary to ensure that the strain rates (and, eventually, strains) are identified with the same material particle. The strains are obtained over the solution field by integrating the transformed strain rates. Three extensional strain components finally result: One is parallel to the flowlines, another is perpendicular in the axial plane, and the third is circumferential. The resultant shear lies in the axial plane with reference to axes parallel and normal to the flowlines. Because of the rotating reference axes through the integration region, the computed shear values are not strains, but are referred to as "accumulated shear." The transformation and integration equations are given in Ref 8.

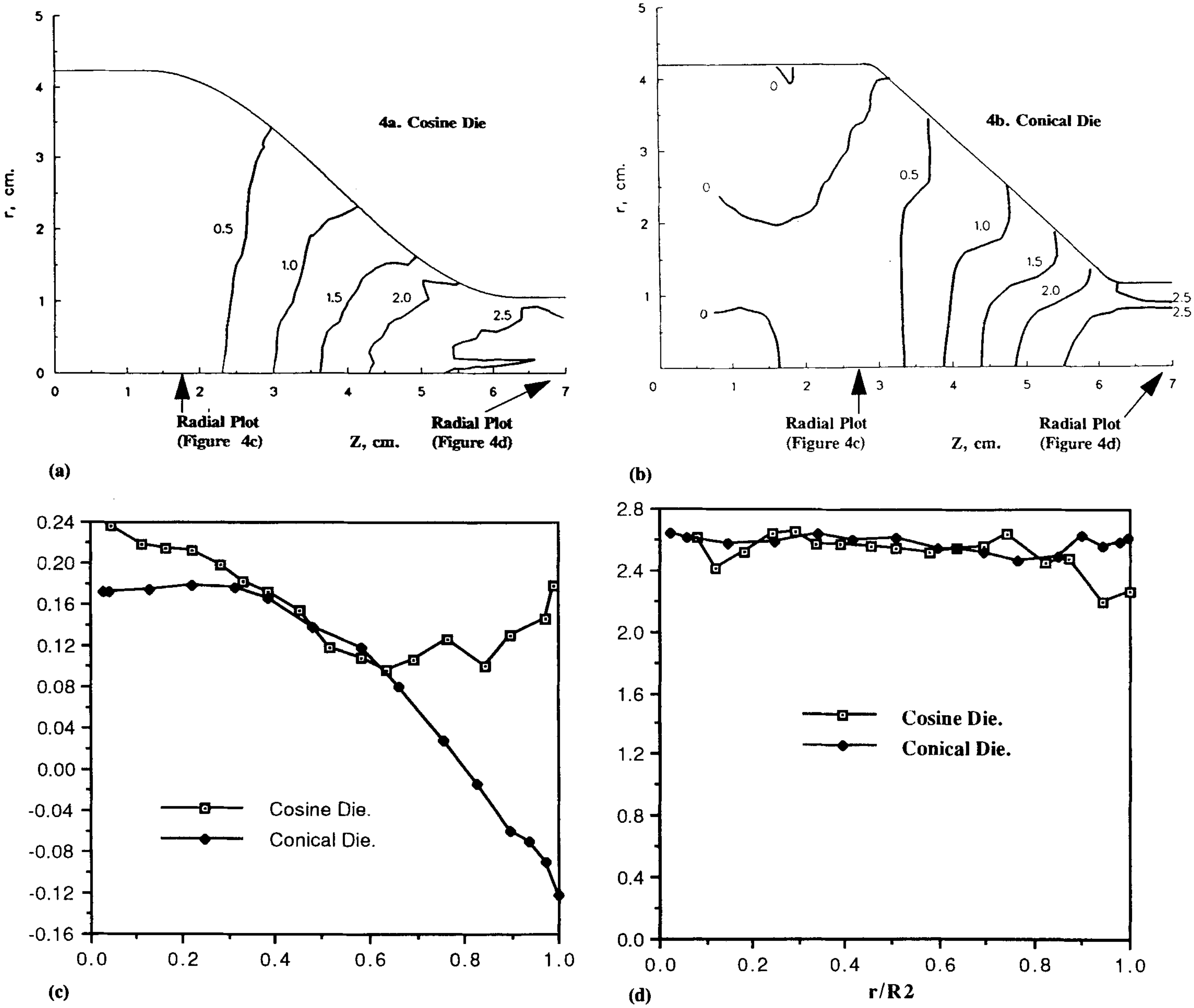


**Fig. 4** Strain parallel to flowlines. (a) Cosine die. (b) Conical die. (c) $R1$, die radius at entry corner. $z$ = 1.8 cm (cosine die); $z$ = 2.8 cm (conical die). (d) $R2$, die radius at exit. $z$ = 7.0 (both dies)

## 4. Experimental Procedure

The experiment used a billet of commercial Alcoa 1100 aluminum, 8.9 cm in diameter and 10.2 cm long. The billet was machined initially into two equal half cylinders.

A grid consisting of 1.27 mm squares was stamped on the axial plane of one of the halves using high-temperature black ink (Phillips Process Company, Rochester, New York) and a special stamping fixture (Federal Stamp and Seal Company, Atlanta, Georgia) to ensure grid alignment. The billet was welded together at the seam separating the two halves and was filed smooth.

Extrusion was performed in a $4.4 \times 10^6$ N extrusion press with a ram speed of 46 cm/min. The lubricant was a mixture of oil and tin. The billet assembly and extrusion tooling were preheated to 350 °C. The extrusion was arrested and the partially extruded billet removed. The two halves of the billet were separated to expose the deformed grid (Fig. 2). The comparable billet from the extrusion through the conical die (Ref 8) is shown in Fig. 3. For the conical die, $\alpha = 45°$. The internal grid procedure was similar to that used by Yang et al. (Ref 6) to examine flow in noncircular extrusions. It permitted study of aluminum directly; many previous extrusion studies (Ref 1, 7) have used lead.

## 5. Computations

Computations included numerical approximations for flowlines, strain-rate calculations (Eq 2-5), strain-rate transfor-

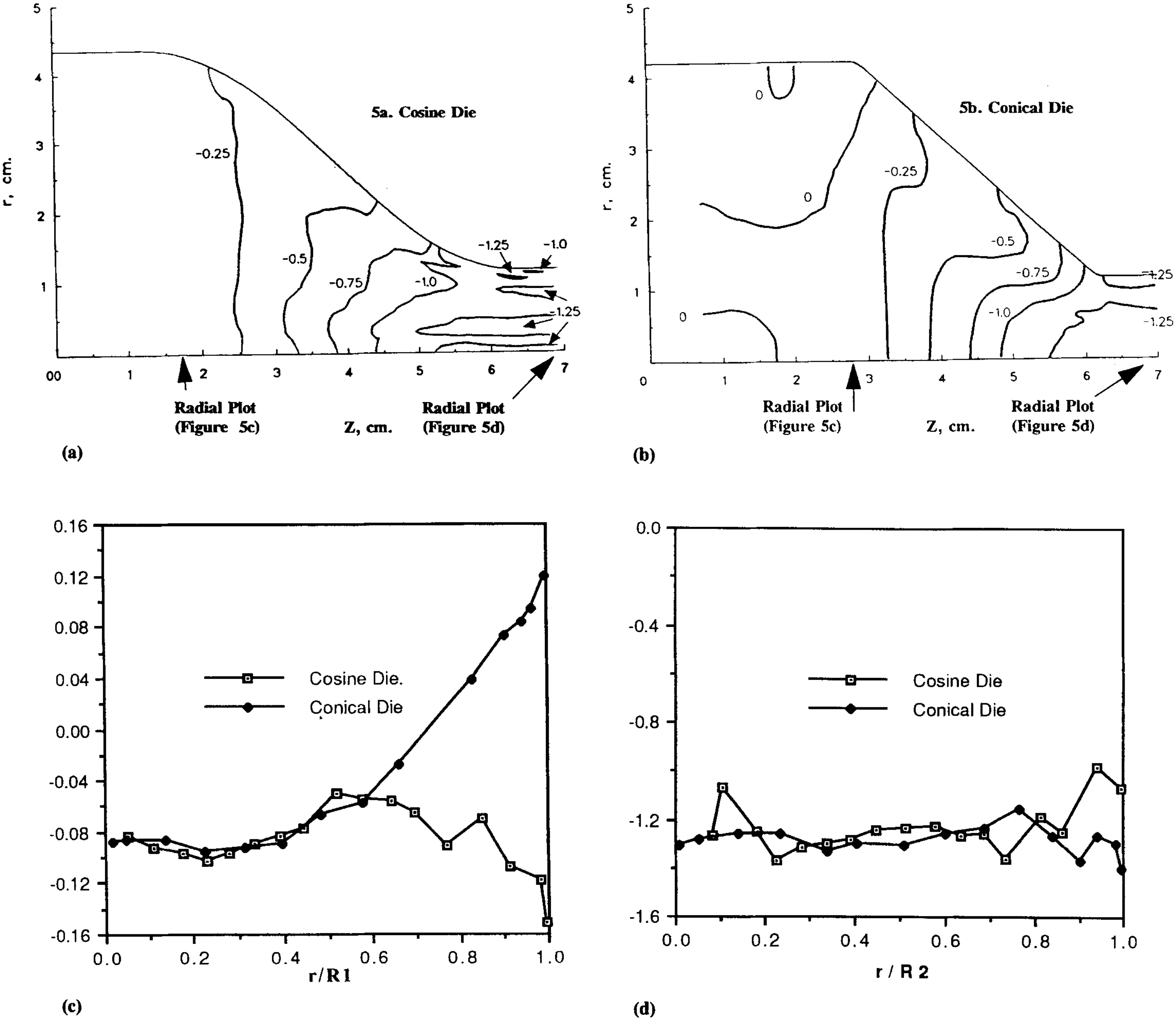


**Fig. 5** Strain perpendicular to flowlines. (a) Cosine die. (b) Conical die. (c) $R1$, die radius at entry corner. $z = 1.8$ cm (cosine die); $z = 2.8$ cm (conical die). (d) $R2$, die radius at exit. $z = 7.0$ (both dies)

mations, and integration for strains. The various computations were essentially the same as those used previously (Ref 8), with one exception.

Smooth spline functions were used to approximate 17 flowlines that were spaced about equally from near the axis to the outer die wall. Discrete data values were obtained using a digitizing tablet and a photograph (enlarged 3.5×) of the deformed grid area. A spline was used to approximate the data for each flowline. The spline construction procedure was identical to that used previously for the flowlines.

Solution stations were established over the region using 40 radial mesh lines placed 1.7 mm apart in the axial direction. These lines were placed numerically, extending radially from the die axis to the outermost flowline. Intersections of radial mesh lines and flowlines were used as locations for all subsequent calculated values and for the results. Modified smooth splines (Ref 10) were used along these lines to approximate $\phi$ and $(1/r)(\partial\varphi/\partial r)$, as previously, with symmetry forced across the billet axis. Unlike the common smooth spline used in Ref 8, the modified smooth spline allows specified second derivatives to be used at the beginning and end of the spline. This avoids the presumed, and often erroneous, "zero" second derivatives at the boundaries of the common smooth spline. The estimated errors were 0.5 $cm^3$/s for all data for $\phi$ and 0.25 $cm^2$/s for all data for $(1/r)(\partial\varphi/\partial r)$.

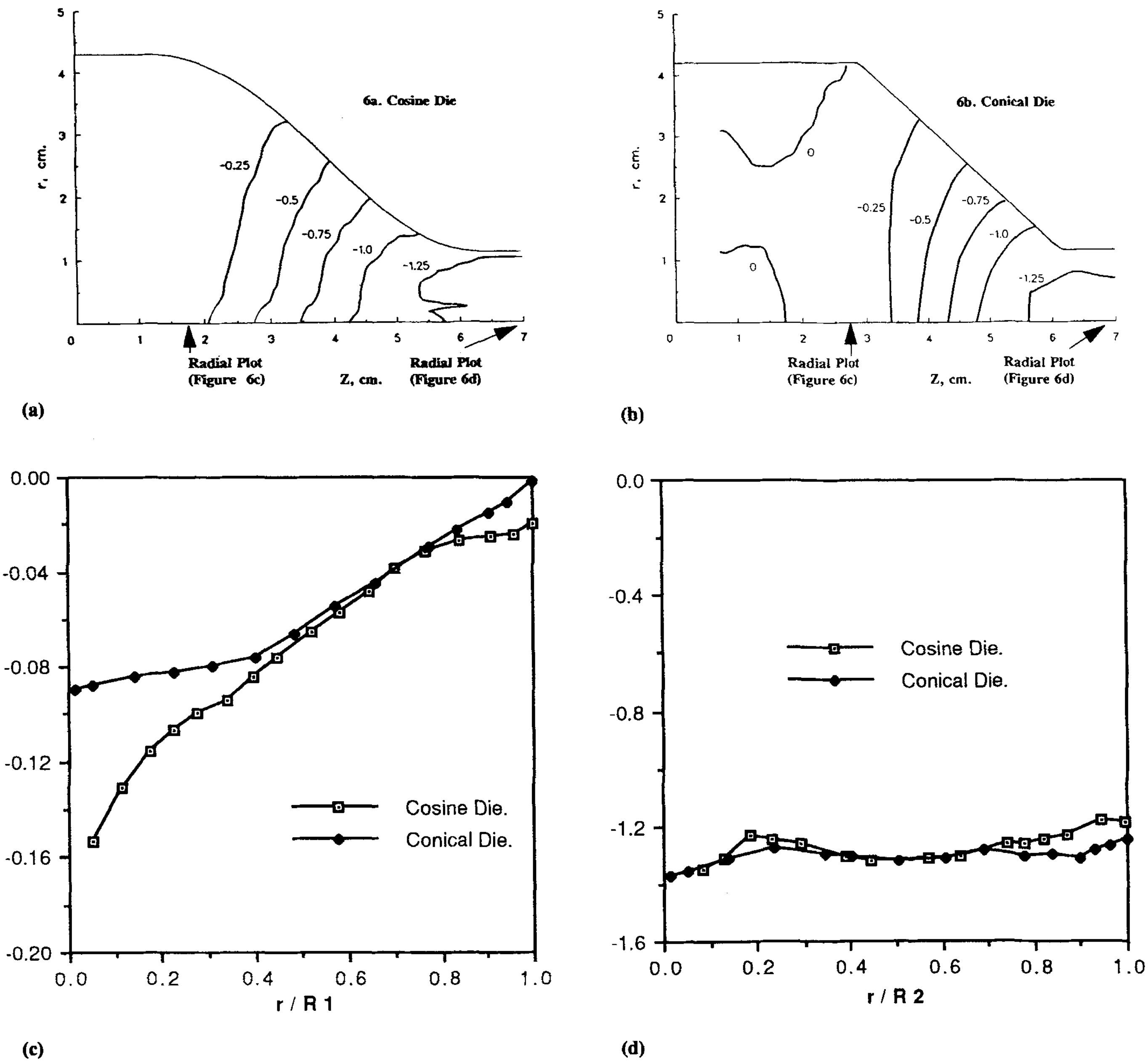


**Fig. 6** Circumferential strain. (a) Cosine die. (b) Conical die. (c) $R1$, die radius at entry corner. $z$ = 1.8 cm (cosine die); $z$ = 2.8 cm (conical die). (d) $R2$, die radius at exit. $z$ = 7.0 (both dies)

Straightforward calculations were used for the strain-rate transformations. Simpson's rule was used to integrate for the strain values.

## 6. Results

The strain results are shown as contour plots over the solution region in Fig. 4 to 7. Plots along significant radial lines are also given. Each figure includes the comparable strain results for the billet extruded through the conical die.

The results for strain parallel to the flowlines (Fig. 4) are similar for both dies, with the exception of the entry region near the die wall. In this region the conical die produces compressive strain. This can be seen to the left of the "zero" contour curve, which passes below that region (Fig. 4a). Compressive strain is also seen in the radial plot (Fig. 4c).

Strain results normal to the flowlines (Fig. 5) are also similar for the two regions, except in the same die wall region at the entry corner. The conical die produces tensile strain in this region, whereas the cosine die produces compressive strain throughout the extrusion region. As seen in Fig. 5(c), the tensile strain at the die wall is the highest magnitude along the die radius.

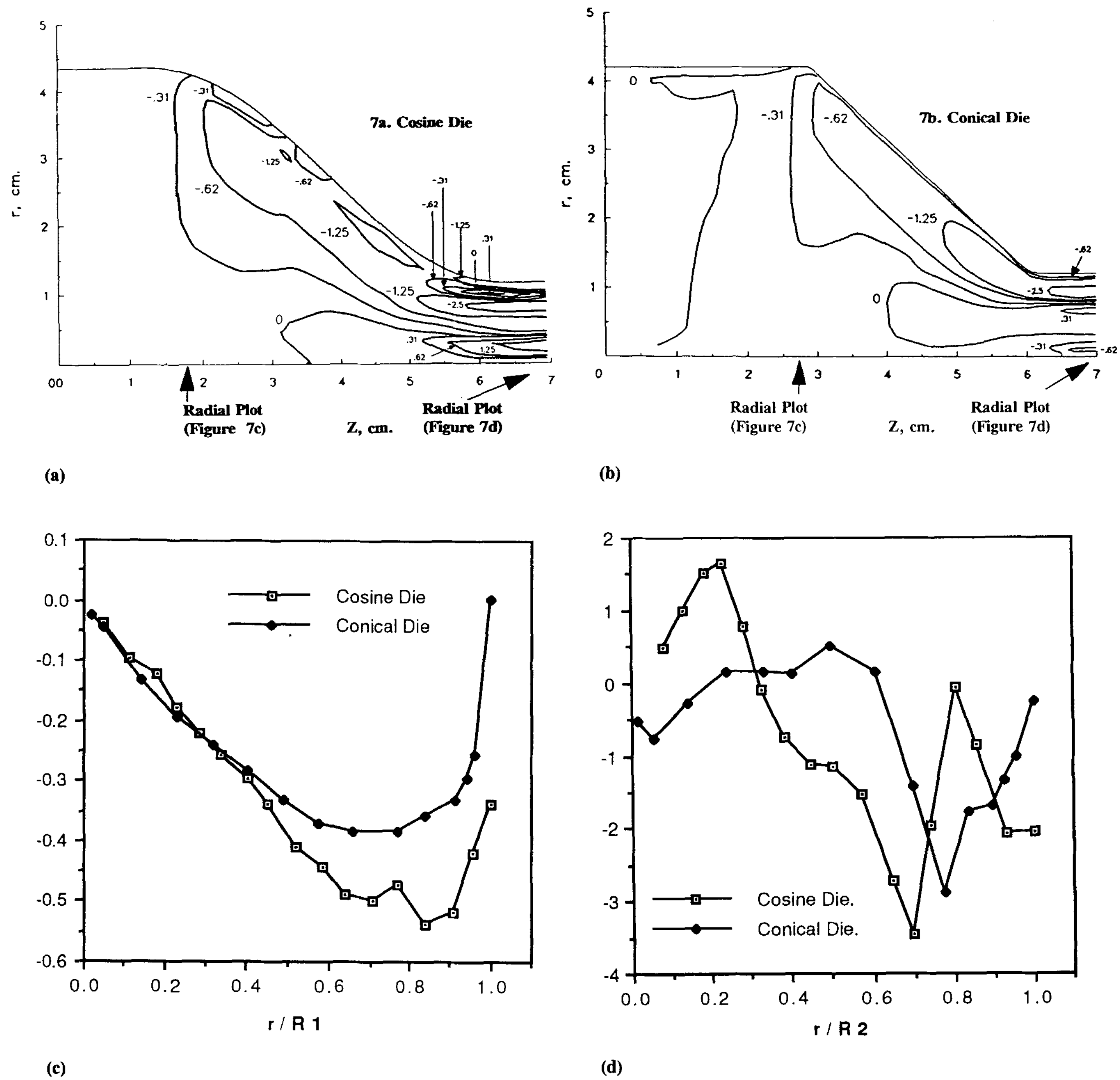


**Fig. 7** Accumulated shear. (a) Cosine die. (b) Conical die. (c) $R1$, die radius at entry corner. $z$ = 1.8 cm (cosine die); $z$ = 2.8 cm (conical die). (d) $R2$, die radius at exit. $z$ = 7.0 (both dies)

Circumferential strain results are similar for both dies (Fig. 6). The accumulated shear results are also similar (Fig. 7), although the steep gradients produced by both dies are not quite the same. One difference is a small interior region of reversed sign close to the outer boundary near the exit of the cosine die. Results in that region include values up to 0.345. With the exception of this small region, the results consist of a region of negative shear near the die wall, with the shear becoming positive near the die axis. This behavior is similar for both dies. The region of highest strain magnitude is away from the wall for both dies. In both regions, the shear magnitudes are higher in the cosine die than in the conical die, with the difference being greater in the region near the die axis.

There are two checks for visioplastic results (Ref 8, 10). First, the average circumferential strain at the die exit as computed through the numerical visioplastic solution should equal

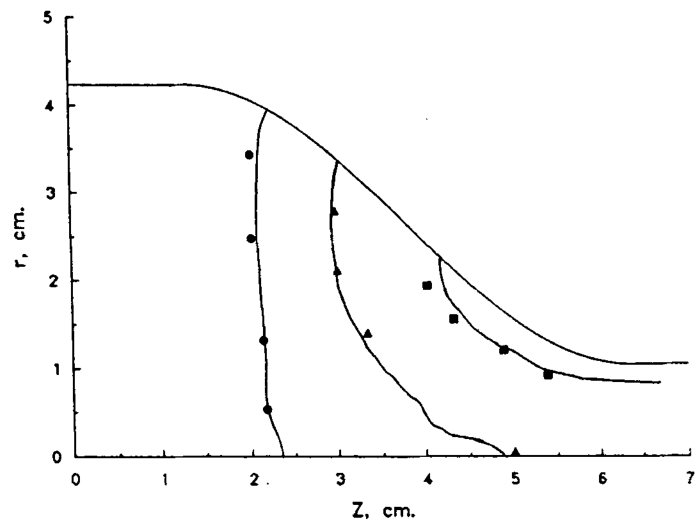


**Fig. 8** Isochronal curves.

the logarithmic circumferential strain. For the cosine die analysis, both values are 1.25. Also, the isochronal curves as computed from elapsed times in the analysis should be approximately the same as isochronals formed by the grid lines that are originally normal to the flowlines. As shown in Fig. 8, most values obtained from the grid photo are close for the three computed isochronals.

## 7. Discussion

These results point to an advantage of the cosine die for the extrusion of powder aluminum. This advantage is found in the entry corner of the die. In this region, the conical die produces tensile strain perpendicular to the flow and compressive strain parallel to the flow. This reversal of the strain is similar to a "dead zone" (Ref 3, 11), where the material does not continue in the regular stream path, and is produced by 45° conical dies at various extrusion ratios (Ref 9). Radial plots of strain in the entry corner region show this behavior most clearly (Fig. 4c and 5c). The reversal of strain signs is absent in the cosine die.

A material characterization study of powder aluminum (Ref 12) showed that the tensile strength and properties parallel to the extrusion direction (the direction of material flow) were improved during biaxial testing in which compressive loading occurred normal to the direction of the elongated particles, or ligaments, that constitute the laminar structure of the material. This was considered to be due to the comparatively better adhesion of the layers and to the clamping effect of the transverse normal load.

To compare the extrusion results, the tensile strain perpendicular to the flowlines in the conical die would produce particle and ligament separations, inhibiting the ability of the powder material to form a cohesive laminar structure in the die wall region. These internal effects would encourage failure of the material during extrusion. The cosine die is found to keep constant signs throughout the solution region (near the die wall), with tensile strains parallel to the flow and compressive strains normal to the flowlines. The compressive transverse strains indicate a desirable clamping effect that improves the strength of the material during extrusion.

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